UCSD/PTH 00-16

Bound on $\cos\alpha$ from exclusive weak radiative B decays

Dan Pirjol

Department of Physics, University of California at San Diego, La Jolla, CA 92093

(February 1, 2008)

Abstract

We present a bound on the weak phase α from isospin-breaking effects in weak radiative decays, which requires the CP-averaged branching ratios for the weak radiative decays $B^\pm \to \rho^\pm\gamma$, $B^0 \to \rho^0/\omega\gamma$, $B \to K^*\gamma$ and the photon energy spectrum in $B \to \gamma\ell\nu_\ell$. We carefully identify all sources of isospin breaking, which could possibly mask information about the CKM parameters. They are introduced by diagrams with photon bremsstrahlung off the spectator quark and diagrams with annihilation penguin topologies. The former can be eliminated by combining $B \to \rho\gamma$ and $B \to K^*\gamma$ data, whereas the latter effects are OZI-suppressed and can be controlled by measuring also the $B_s \to \rho(\omega)\gamma$ modes. The resulting bound excludes values of α around 90°, provided that the combined ratio $\mathcal{B}(B^\pm \to \rho^\pm\gamma)/\mathcal{B}(B^0 \to \rho^0/\omega\gamma) \times \mathcal{B}(B^0 \to K^{*0}\gamma)/\mathcal{B}(B^\pm \to K^{*\pm}\gamma)$ is found to be different from 1.

The determination of the weak phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is one of the main goals of physics studies at B factories. The angle $\beta = -\text{Arg}\ (V_{td})$ will be measured from the time-dependent CP asymmetry in $B \to J/\psi K_S$ decays [1]. The phase $\gamma = \text{Arg}\ (V_{ub}^*)$ can be determined precisely from time-independent measurements of rates for $B^\pm \to K^\pm D$. Alternatively, it can be extracted using SU(3) symmetry from a combination of $B^+ \to K\pi$ and $B^+ \to \pi^+\pi^0$ decays [2]. On the other hand, an extraction of the weak phase $\alpha = \pi - \beta - \gamma$ could prove more challenging. The standard method for measuring α from time-dependent measurements in $B \to \pi\pi$ decays is hampered by the need to perform a demanding isospin analysis [3].

We propose in this paper a novel method for constraining the weak phase α from isospin breaking effects in exclusive radiative B decays. This method makes use of the charge-averaged rates for the weak radiative modes $B^{\pm,0} \to \rho^{\pm,0}\gamma$, $B^{\pm,0} \to K^{*\pm,0}\gamma$ and the photon spectrum in the radiative leptonic decay $B^\pm \to \gamma e\nu$. All these decays are expected to have branching ratios of the order of 10^{-6}, which is comparable to the expected rates for the $B \to \pi\pi$ modes needed for the traditional method for determining α. On the positive side, the method presented here requires no tagging or time-dependent measurements. Also, the detection efficiency for these modes is very good, which is known to represent a problem with $B^0 \to \pi^0\pi^0$. On the negative side, this method requires a binning of the photon spectrum in $B^\pm \to \gamma e\nu$, at a value of the photon energy equal to that in $B \to \rho\gamma$ decay ($E_\gamma = 2.6$ GeV).

Exclusive radiative B decays are notorious for being plagued with hard-to-calculate long-distance contributions, arising from charm- and up-quark loop diagrams [5–13]. We propose here to eliminate some of these unknown contributions, by combining $B \to \rho(\omega)\gamma$ with $B \to K^*\gamma$ data. The remaining long-distance amplitudes are OZI-suppressed and hence can be argued to be small. Their size can be controlled by measuring also the $B_s \to \rho(\omega)\gamma$ modes. On the theoretical side, we rely on the fact that the dominant long-distance amplitude, connected with the weak annihilation graph, is exactly calculable to the leading order in an expansion in $1/E_\gamma$, in terms of data observable in $B \to \gamma e\nu$ decays [14]. A somewhat related method for determining $\cos\alpha$ has been proposed recently in [15] in terms of the exclusive decay $B \to \pi e^+e^-$ on which certain cuts have been imposed. Unfortunately, the small rate of this mode makes this determination very challenging from an experimental point of view.

We consider weak radiative decays of B mesons into a vector meson within SU(3) symmetry. This decay can proceed either through the direct (short-distance) penguin transition $\bar{b} \to \bar{s}(\bar{d})\gamma$ or through the four-quark weak decay $\bar{b} \to \bar{q}_1 q_2 \bar{s}$ (long-distance) with the photon attaching to any internal quark line. The relevant decay amplitudes can be written as linear combinations of a few amplitudes, each corresponding to a possible quark diagram (see Fig. 1). The dominant amplitude is induced by the photon penguin $\mathcal{Q}_7$ and is denoted with P_t; the gluonic penguin $\mathcal{Q}_8$ is responsible for the amplitudes $G^{(i)}$, where the index i distinguishes between diagrams with the photon attaching to the spectator quark in the B meson ($i = 2$) or the remaining quark lines ($i = 1$). The remaining amplitudes are induced by the four-quark operators in the weak hamiltonian and consist of the weak annihilation (WA) amplitude A, W-exchange amplitude E, penguin-type amplitudes with internal u and c quarks $P_u^{(i)}$, $P_c^{(i)}$ and annihilation-type penguin amplitudes $PA_u^{(i)}$, $PA_c^{(i)}$. In the latter four amplitudes we distinguish again between the spectator vs. non-spectator attachments of the photon to the quark lines in the diagram. This decomposition in terms of graphical

amplitudes is equivalent to a more conventional SU(3) analysis [16], and we have checked that the two approaches give the same results.

The $B \to \rho\gamma$ decay amplitudes are given in terms of graphical amplitudes as (for each photon helicity $\lambda = L, R$).

$$
\begin{aligned}
A(B^- \to \rho^- \gamma_\lambda) &= \lambda_u^{(d)} (P_{u\lambda}^{(1)} + Q_u P_{uc\lambda}^{(2)} + A_\lambda) + \lambda_c^{(d)} (P_{c\lambda}^{(1)} + Q_u P_{c\lambda}^{(2)}) \qquad (1)\\
&\quad + \lambda_t^{(d)} (P_{t\lambda} + G_\lambda^{(1)} + Q_u G_\lambda^{(2)})\\
\sqrt{2} A(\bar{B}^0 \to \rho^0 \gamma_\lambda) &= \lambda_u^{(d)} (P_{u\lambda}^{(1)} + Q_d P_{u\lambda}^{(2)} - E_\lambda - PA_{u\lambda}) + \lambda_c^{(d)} (P_{c\lambda}^{(1)} + Q_d P_{c\lambda}^{(2)}) \qquad (2)\\
&\quad + \lambda_t^{(d)} (P_{t\lambda} + G_\lambda^{(1)} + Q_d G_\lambda^{(2)} + PA_{c\lambda}) \,.
\end{aligned}
$$

The CKM factors are defined as $\lambda_q^{(q')} = V_{qb} V_{qq'}^*$.

In the Standard Model, the amplitudes (1), (2) are dominated by the short-distance penguin amplitude coupling to a left-handed photon P_{tL}. The remaining long-distance amplitudes are typically about 5-10% of the leading P_{tL} amplitude. Therefore we will write (1), (2) by expanding in the small ratios of long-distance/short-distance contributions as

$$
\begin{aligned}
A(B^- \to \rho^- \gamma_L) &= \lambda_t^{(d)} p_L \left(1 + \left| \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*} \right| e^{-i(\beta+\gamma)} \left(\varepsilon_A + \varepsilon_{uc}^{(1)} + \frac{2}{3} \varepsilon_{uc}^{(2)} \right) \right) \qquad (3)\\
\sqrt{2} A(\bar{B}^0 \to \rho^0 \gamma_L) &= \lambda_t^{(d)} p_L \, (1 - \varepsilon_{\mathrm{sp}} + \varepsilon_{PA_c} + \qquad (4)\\
&\quad \left| \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*} \right| e^{-i(\beta+\gamma)} \left(-\varepsilon_E - \varepsilon_{PA_{uc}} + \varepsilon_{uc}^{(1)} - \frac{1}{3} \varepsilon_{uc}^{(2)} \right) \Bigg) \,,
\end{aligned}
$$

where the unitarity of the CKM matrix $\lambda_u^{(q)} + \lambda_c^{(q)} + \lambda_t^{(q)} = 0$ has been used to eliminate the terms proportional to $\lambda_c^{(q)}$. We denoted here $p_L = P_{tL} + G_L^{(1)} + \frac{2}{3} G_L^{(2)}$ and introduced the small complex quantities

$$
(\varepsilon_{\mathrm{sp}} \,, \varepsilon_A \,, \varepsilon_{uc}^{(i)} \,, \varepsilon_{PA_c} \,, \varepsilon_{PA_{uc}}) = \frac{1}{p_L} (G_L^{(2)} - P_{cL}^{(2)} \,, A_L \,, P_{uL}^{(i)} - P_{cL}^{(i)} \,, PA_{cL} \,, PA_{uL} - PA_{cL}) \,. \qquad (5)
$$

Let us consider the isospin-violating ratio of CP-averaged decay rates

$$
R_1 \equiv \frac{\tau_{B^0}}{\tau_{B^\pm}} \frac{\mathcal{B}(B^\pm \to \rho^\pm \gamma)}{2\mathcal{B}(B^0 \to \rho^0 \gamma)} \,. \qquad (6)
$$

In the absence of the long-distance contributions, the amplitudes appearing in this ratio are dominated by the penguin P_t, and the value of the ratio is unity. The deviation of R_1 from 1 is due to interference between the long-distance and short-distance amplitudes.

Expanding in the small parameters ε_i and keeping only the linear terms, the isospin-violating ratio R_1 is given by

$$
R_1 = 1 + 2\mathrm{Re}\,(\varepsilon_{\mathrm{sp}} - \varepsilon_{PA_c}) - 2 \left| \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*} \right| \cos\alpha \; \mathrm{Re} \left(\varepsilon_A + \varepsilon_E + \varepsilon_{PA_{uc}} + \varepsilon_{uc}^{(2)} \right) + \mathcal{O}(\varepsilon_i^2) \,. \qquad (7)
$$

In principle there are also long-distance amplitudes contributing to decays into right-handed photons. However they enter the ratio R_1 only quadratically such that they were not written explicitly.

The long-distance amplitudes appearing in (7) have been estimated using a variety of approaches [5–13,17,18,20,21]. Although the detailed numerical predictions differ somewhat, a certain hierarchy of sizes can be discerned [14]. The dominant amplitude is the WA amplitude coupling to a left-handed photon A_L. The W-exchange amplitude E is both color- and charge-suppressed relative to A_L by about a factor of 10 [6–8] so that it will be neglected in the following.

The remaining long-distance amplitudes in (7) are considerably more difficult to calculate. The annihilation-type penguin amplitudes $PA_{u,c}$ can be argued to be OZI-suppressed and hence very small. The spectator-type amplitude $\varepsilon_{\rm sp}$, arising from diagrams with insertions of the gluonic penguin $G_L^{(2)}$ and diagrams with charm loops $P_{cL}^{(2)}$ in which the photon couples to the spectator quark, is not necessarily small. Certain contributions of this type have been computed in [20] within the SM, and they were found to be typically of the order of 5% of the short-distance amplitude. It is conceivable that this amplitude could be significant. For example, in certain scenarios of new physics involving an enhanced gluonic penguin [22], this amplitude can be greatly enhanced [23].

It has been proposed in the literature [7,12,18,19] to use a measurement of the isospin-violating ratio R_1 in order to extract information on the CKM parameters $(\rho\,,\eta)$. Given the smallness of the expected effect, it is clear than even a value as small as $\mathrm{Re}\,\varepsilon_{\rm sp} = 0.05$ in (7) could introduce a significant uncertainty of 50% in the constraint on $\cos\alpha$ (we used here the model estimate $\varepsilon_\rho \simeq 0.12$, see below).

In this Letter we point out that the long-distance amplitude $\varepsilon_{\rm sp}$ can be eliminated with the help of data on weak radiative $B \to K^*\gamma$ decays. The corresponding decay amplitudes can be written, in the SU(3) limit, in terms of the same graphical amplitudes as those appearing in (1), (2)

$$\begin{aligned} A(B^- \to K^{*-}\gamma_\lambda) &= \lambda_t^{(s)}(P_{t\lambda} + G_\lambda^{(1)} - P_{c\lambda}^{(1)} + Q_u G_\lambda^{(2)} - Q_u P_{c\lambda}^{(2)}) \\ \sqrt{2}A(\bar B^0 \to K^{*0}\gamma_\lambda) &= \lambda_t^{(s)}(P_{t\lambda} + G_\lambda^{(1)} - P_{c\lambda}^{(1)} + Q_d G_\lambda^{(2)} - Q_d P_{c\lambda}^{(2)})\,. \end{aligned} \tag{8}$$

We neglected here terms proportional to $\lambda_u^{(s)}$ which are Cabibbo suppressed. Introducing the ratio of CP-averaged branching ratios for the $B \to K^*\gamma$ modes one finds

$$R_2 \equiv \frac{\tau_{B^\pm}}{\tau_{B^0}} \frac{\mathcal{B}(B^0 \to K^{*0}\gamma)}{\mathcal{B}(B^\pm \to K^{*\pm}\gamma)} = 1 - 2\mathrm{Re}\,\varepsilon_{\rm sp} + \mathcal{O}(\varepsilon_i^2)\,, \tag{9}$$

where we expanded again in the small parameters ε_i and kept only the linear terms.

The CLEO Collaboration recently measured the branching ratios for the exclusive $B \to K^*\gamma$ modes [24]

$$\mathcal{B}(B^\pm \to K^{*\pm}\gamma) = (3.76^{+0.89}_{-0.83} \pm 0.28) \times 10^{-5} \tag{10}$$

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.55^{+0.72}_{-0.68} \pm 0.34) \times 10^{-5}\,. \tag{11}$$

Adding the statistical and systematic errors in quadrature one finds

$$R_2 = 1.29 \pm 0.37\,, \tag{12}$$

where we used the lifetime ratio of charged and neutral B mesons $\tau(B^\pm)/\tau(B^0) = 1.066 \pm 0.024$ [25]. Although the error in this determination is still large, this leaves open the

possibility of a significant long-distance amplitude $\varepsilon_{\rm sp}$. We propose therefore to eliminate $\varepsilon_{\rm sp}$ by using instead of R_1 the following combined isospin-violating ratio

$$R_\rho \equiv R_1 R_2 = \frac{\mathcal{B}(B^\pm \to \rho^\pm\gamma)}{2\mathcal{B}(B^0 \to \rho^0\gamma)} \cdot \frac{\mathcal{B}(B^0 \to K^{*0}\gamma)}{\mathcal{B}(B^\pm \to K^{*\pm}\gamma)} = 1 - 2{\rm Re}\,\varepsilon_{PA_c} + 2{\rm Re}\,\varepsilon_\rho \cos\alpha + \mathcal{O}(\varepsilon_i^2)\,. \quad (13)$$

We will neglect in the following the contribution of the OZI-suppressed amplitude ε_{PA_c}. A measure of the validity of this approximation could be obtained by measuring the rate for $B_s \to \rho^0\gamma$. In the SU(3) limit the amplitude for this decay is given by $\sqrt{2}A(\bar B_s \to \rho^0\gamma) = \lambda_u^{(s)}(E + PA_{uc}) - \lambda_t^{(s)} PA_c$. Neglecting the contribution of the CKM suppressed term $\lambda_u^{(s)}(E + PA_{uc})$ one finds the following upper bound

$$|\varepsilon_{PA_c}|^2 \le \frac{2\Gamma(B_s \to \rho^0\gamma)}{\Gamma(B^\pm \to K^{*\pm}\gamma)}\,. \quad (14)$$

The deviation of R_ρ from unity is proportional to the quantity

$$\varepsilon_\rho \simeq -\left|\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\right| |\varepsilon_A| e^{i\phi_A} \quad (15)$$

where we neglected the other long-distance amplitudes ε_i, $i = E, PA_{uc}, P_{uc}^{(2)}$. Leaving the strong phase of the WA amplitude ϕ_A completely arbitrary gives an inequality on $\cos\alpha$ provided that $R_\rho \neq 1$

$$|\cos\alpha| \ge |R_\rho - 1|/(2|\varepsilon_\rho|)\,. \quad (16)$$

This excludes a region in $\cos\alpha$ around $\alpha = 90^\circ$, which is of interest since this value lies within the range favored by present global fits of the unitarity triangle [26].

The only ingredient missing for turning this into an observable prediction is an estimate for ε_ρ. We propose to use for this a combination of data on $B \to \gamma e\nu$ and $B^\pm \to K^{\pm *}\gamma$ decays. To leading order of an expansion in the small parameter $1/E_\gamma$, the dominant WA amplitudes A_λ are given exactly by the factorized result [14]

$$A_{L,R} = -\frac{G_F}{\sqrt{2}}(C_2 + \frac{C_1}{N_c}) e m_\rho f_\rho \left(f_B + E_\gamma(f_A(E_\gamma) \mp f_V(E_\gamma))\right) + \mathcal{O}(\frac{\Lambda^2}{E_\gamma^2})\,. \quad (17)$$

$C_{1,2}$ are Wilson coefficients appearing in the weak nonleptonic Hamiltonian [27]. The form-factors $f_{V,A}(E_\gamma)$ parametrize the $B \to \gamma e\nu$ decay, and are defined by

$$\begin{aligned}\frac{1}{\sqrt{4\pi\alpha}}\langle\gamma(q,\epsilon_\lambda)|\bar q\gamma_\mu(1-\gamma_5)b|\bar B(v)\rangle = \\ i\varepsilon(\mu,\epsilon_\lambda^*,v,q) f_V(E_\gamma) + [\epsilon_{\lambda\mu}^*(v\cdot q) - q_\mu(\epsilon_\lambda^*\cdot v)] f_A(E_\gamma)\,,\end{aligned} \quad (18)$$

where v denotes the B meson velocity ($p_B = m_B v$). They are in principle measurable from the doubly differential spectrum of the radiative leptonic decay $B^\pm \to \gamma e\nu$. Model calculations of these form factors suggest that the left-handed amplitude A_L in B^- decay is about 30% of the short-distance penguin amplitude P_{tL} [6,7,12,14] and is enhanced by about a factor of 7 relative to the right-handed amplitude A_R. A certain simplification is

obtained by considering the form factors $f_{V,A}(E_\gamma)$ in an expansion in powers of $1/E_\gamma$. The leading terms in this expansion are related as [28]

$$f(E_\gamma) \equiv f_V^{(B^\pm)}(E_\gamma) = \pm f_A^{(B^\pm)}(E_\gamma) + \mathcal{O}(\frac{\Lambda^2}{E_\gamma^2})\,. \tag{19}$$

This implies that, to leading twist, the combination $|V_{ub}|\cdot|f(E_\gamma)|$ can be extracted from a measurement of the (more accessible experimentally) photon energy spectrum in $B^\pm\to\gamma\ell\nu_\ell$ decay. Combining this with the rate for $B^\pm\to K^{*\pm}\gamma$, the parameter $|\varepsilon_\rho|$ can be determined up to corrections of order $\Lambda/E_\gamma\simeq 13\%$.

$$|\varepsilon_\rho|^2 = \frac{\frac{d}{dE_\gamma}\Gamma(B^\pm\to\gamma e\nu)|_{E_\gamma=2.6\,GeV}}{\Gamma(B^\pm\to K^{*\pm}\gamma)}\cdot\frac{3(2\pi)^2(C_2+\frac{C_1}{3})^2 m_\rho^2 f_\rho^2}{m_B^2(m_B-2E_\gamma)}\left(1-\frac{f_B}{2E_\gamma f(E_\gamma)}\right)^2\,. \tag{20}$$

Model calculations of the formfactors $f_{V,A}(E_\gamma)$ suggest that $|\varepsilon_A|\simeq 0.3$ [7,12,29], which gives $|\varepsilon_\rho|\simeq 0.12$ (we used here $|V_{ub}|/|V_{td}|\simeq 0.4$ [30]). The last factor in (20) is a 7% correction and its calculation requires theoretical input about the form factors. In addition, the result (17) implies that, to leading twist, the strong phase ϕ_A vanishes. This turns the bound in (16) into an identity, and the bound on $|\cos\alpha|$ into a determination of this parameter. Alternatively, the strong phase ϕ_A can be completely eliminated by combining R_ρ with a measurement of the CP asymmetry

$$A_{CP}^{(\rho)} = \frac{\Gamma(B^-\to\rho^-\gamma)-\Gamma(B^+\to\rho^+\gamma)}{\Gamma(B^-\to\rho^-\gamma)+\Gamma(B^+\to\rho^+\gamma)} = 2|\varepsilon_\rho|\sin\alpha\sin\phi_A + \mathcal{O}(\varepsilon_i^2)\,. \tag{21}$$

Similar results are obtained for the combined ratio involving the decay $B\to\omega\gamma$. The $B\to\omega\gamma$ amplitude depends on additional graphical amplitudes $S_{u,c}$ connected with decays into an SU(3) singlet. They are similar to the amplitudes $PA_{u,c}$ shown in Fig. 1(d), except that the photon attaches to the other quark lines in the diagram. Assuming ideal mixing in the (ω,ϕ) system, one has

$$\begin{aligned}\sqrt{2}A(\bar B^0\to\omega\gamma_\lambda) &= -\lambda_u^{(d)}(P_{u\lambda}^{(1)}+Q_dP_{u\lambda}^{(2)}+E_\lambda+\frac{1}{3}PA_{u\lambda}+\frac{2}{3}S_{u\lambda}) \\ &\quad -\lambda_c^{(d)}(P_{c\lambda}^{(1)}+Q_dP_{c\lambda}^{(2)}+\frac{1}{3}PA_{c\lambda}+\frac{2}{3}S_{c\lambda})-\lambda_t^{(d)}(P_{t\lambda}^{(1)}+G_\lambda^{(1)}+Q_dG_\lambda^{(2)})\,.\end{aligned} \tag{22}$$

Defining again a combined ratio of decay rates as in (13) one finds to linear order in the small parameters ε_i

$$R_\omega \equiv \frac{\mathcal{B}(B^\pm\to\rho^\pm\gamma)}{2\mathcal{B}(B^0\to\omega\gamma)}\cdot\frac{\mathcal{B}(B^0\to K^{*0}\gamma)}{\mathcal{B}(B^\pm\to K^{*\pm}\gamma)} = 1-2\mathrm{Re}\,(\frac{1}{3}\varepsilon_{PA_c}+\frac{2}{3}\varepsilon_{S_c})+2\mathrm{Re}\,\varepsilon_\omega\cos\alpha+\mathcal{O}(\varepsilon_i^2)\,. \tag{23}$$

We denoted here $\varepsilon_\omega = -|V_{ub}V_{ud}^*|/|V_{tb}V_{td}^*|(\varepsilon_A-\varepsilon_E+\varepsilon_{uc}^{(2)}-\frac{1}{3}\varepsilon_{PA_{uc}}-\frac{2}{3}\varepsilon_{S_{uc}})$. Keeping again only the dominant WA long-distance amplitude ε_A gives that the two ratios (13), (23) are equally sensitive to $\cos\alpha$: $\varepsilon_\omega\simeq\varepsilon_\rho$.

Using SU(3) symmetry one can give an upper bound on the combination of unknown long-distance amplitudes in (23), similar to (14). Neglecting a Cabibbo-suppressed term

proportional to $\lambda_u^{(s)}$, the decay amplitude for $\bar{B}_s \to \omega\gamma$ is given by $\sqrt{2}A(\bar{B}_s \to \omega\gamma) = -\lambda_t^{(s)}(\frac{1}{3}PA_c + \frac{2}{3}S_c)$, which gives the inequality

$$|\frac{1}{3}\varepsilon_{PA_c} + \frac{2}{3}\varepsilon_{S_c}|^2 \le \frac{2\Gamma(B_s \to \omega\gamma)}{\Gamma(B^\pm \to K^{*\pm}\gamma)}\,. \tag{24}$$

We turn now to the theoretical uncertainties of this method. The neglected quadratic terms in (13) and (23) can be expected to be of the order of 1-2%. A similar contribution is expected from the OZI-suppressed terms proportional to $\varepsilon_{PA_c}, \varepsilon_{S_c}$ which however could be enhanced by rescattering effects. As explained above, the bounds (14), (24) can be used to control the size of these effects. The SU(3) breaking effects introduce an additional uncertainty as an incomplete cancellation of the spectator amplitude $\varepsilon_{\rm sp}$ in R_ρ and R_ω. However the corresponding effect is linear in both SU(3) breaking and $\varepsilon_{\rm sp}$ and hence very small.

The most important theoretical limitation of this method is probably connected with our ability to compute the long-distance parameters $\varepsilon_{\rho,\omega}$ in (13), (23). It is unlikely that this parameter can be computed to better than 15% accuracy, where the uncertainty comes from higher twist effects in (17) and from the neglected long-distance amplitudes $\varepsilon_i, i = E, PA_{uc}, P_{uc}$.

However, at least in the near future, the statistical errors are likely to dominate the precision of any constraints on α which could be obtained from this method. Some improvement in statistics can be achieved by noting that the two ratios R_ρ and R_ω are equal to a first approximation. Therefore we introduce the combined ratio

$$R_{\rho/\omega} \equiv \frac{\mathcal{B}(B^\pm \to \rho^\pm\gamma)}{\mathcal{B}(B^0 \to \rho^0/\omega\gamma)} \cdot \frac{\mathcal{B}(B^0 \to K^{*0}\gamma)}{\mathcal{B}(B^\pm \to K^{*\pm}\gamma)} \simeq 1 + 2|\varepsilon_\rho| \cos\phi_A \cos\alpha + \mathcal{O}(\varepsilon_i^2) \tag{25}$$

for which all the conclusions derived above for R_ρ and R_ω hold unchanged. For example, a sample of 6×10^8 $B\bar{B}$ pairs (the equivalent of 500 fb^{-1} integrated luminosity) would allow the measurement of this ratio to about 20% accuracy, which would give a determination of $\cos\alpha$ at a $\sigma(\langle\cos\alpha\rangle) \simeq 24\%$ level. Even a much less precise constraint could be useful in eliminating discrete ambiguities on α arising from the method based on time-dependent measurements. We assumed in this estimate typical branching ratios of the order $\mathcal{B}(B^\pm \to \rho^+\gamma) = 1.3 \times 10^{-6}$, $\mathcal{B}(B^0 \to \rho^0\gamma) = \mathcal{B}(B^0 \to \omega\gamma) = 0.7 \times 10^{-6}$ [1], neglected the possible backgrounds and assumed the same reconstruction efficiency for ρ, ω as in the CLEO experiment [24]. Although difficult, such a measurement could be performed in the phase I of the existing PEP-II facility, which is expected to have collected about 8×10^8 $B\bar{B}$ pairs by the end of 2008, or at a hadronic B factory.

In conclusion, we presented a constraint on the weak phase α from isospin-breaking effects in exclusive weak radiative decays of B mesons. This is similar to methods proposed earlier in [7,12,18,15,19] to constrain certain combinations of CKM parameters using similar data. We improve on these methods by including all the relevant isospin-violating long-distance amplitudes. Some of these unknown long-distance amplitudes can be eliminated by combining $B \to \rho(\omega)\gamma$ with $B \to K^*\gamma$ data, and the remaining ones are OZI-suppressed and can be controlled with the help of additional B_s weak radiative decays.

ACKNOWLEDGMENTS

I am grateful to Ahmed Ali, Benjamin Grinstein, Alex Kagan and Soeren Prell for very useful discussions. The research of D. P. is supported in part by the DOE and by a National Science Foundation Grant no. PHY-9457911.

REFERENCES

[1] "The BaBar Physics Book" edited by P. F. Harrison and H. R. Quinn, Report No. SLAC-R-504, October 1998.

[2] M. Gronau, talk given at the 3rd International Conference on B Physics and CP Violation (BCONF99), Taipei, Taiwan, 3-7 Dec 1999, hep-ph/0001317.

[3] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).

[4] J. M. Soares, Nucl. Phys. B**367**, 575 (1991).

[5] E. Golowich and S. Pakvasa, Phys. Lett. B**205**, 393 (1988); E. Golowich and S. Pakvasa, Phys. Rev. D**51**, 1215 (1995).

[6] A. Khodjamirian, G. Stoll and D. Wyler, Phys. Lett. B**358**, 129 (1995).

[7] A. Ali and V. M. Braun, Phys. Lett. B**359**, 223 (1995).

[8] A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, Phys. Lett. B**402**, 167 (1997).

[9] D. Atwood, G. Eilam and A. Soni, Mod. Phys. Lett. A**11**, 1061 (1996).

[10] H. Y. Cheng, Phys. Rev. D**51**, 6228 (1995).

[11] H. Y. Cheng *et al.*, Phys. Rev. D**51**, 1199 (1995).

[12] J. F. Donoghue, E. Golowich, A. V. Petrov and J. M. Soares, Phys. Rev. Lett. **77**, 2178 (1996); J. F. Donoghue, E. Golowich and A. V. Petrov, Phys. Rev. D**55**, 2657 (1997).

[13] D. Atwood, B. Blok and A. Soni, Int. J. Mod. Phys. A**11**, 3743 (1996).

[14] B. Grinstein and D. Pirjol, UCSD report UCSD-PTH-00-03, hep-ph/0002216.

[15] B. Grinstein, D. Nolte and I. Rothstein, Phys. Rev. Lett. **84**, 4545 (2000).

[16] D. Zeppenfeld, Z. Phys. C**8**, 77 (1981).

[17] C. Greub, H. Simma and D. Wyler, Nucl. Phys. B**434**, 39 (1995).

[18] G. Eilam, A. Ioannissian and R. R. Mendel, Z. Phys. C**71**, 95 (1996).

[19] A. Ali, L. T. Handoko and D. London, DESY report DESY 00-088, hep-ph/0006175.

[20] C. E. Carlson and J. Milana, Phys. Rev. D**51**, 4950 (1995).

[21] H. H. Asatryan, H. M. Asatrian and D. Wyler, Phys. Lett. B**470**, 223 (1999).

[22] A. Kagan, Phys. Rev. D**51**, 6196 (1995).

[23] A. Petrov, Phys. Lett. B**399**, 172 (1997).

[24] CLEO Collaboration, T. E. Coan *et al*, CLNS 99/1653, hep-ex/9912057.

[25] The B lifetime working group, `http://home.cern.ch∼claires/lepblife.html`

[26] F. Caravaglios, F. Parodi, P. Roudeau and A. Stocchi, Talk given at 3rd International Conference on B Physics and CP Violation (BCONF99), Taipei, Taiwan, 3-7 Dec 1999, hep-ph/0002171.

[27] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[28] G. Korchemsky, D. Pirjol and T. M. Yan, Phys. Rev. D**61**, 114510 (2000).

[29] G. Eilam, I. Halperin and R. R. Mendel, Phys. Lett. B**361**, 137 (1995).

[30] A. Ali and D. London, Eur. Phys. J. C**9**, 687 (1999).

FIGURES

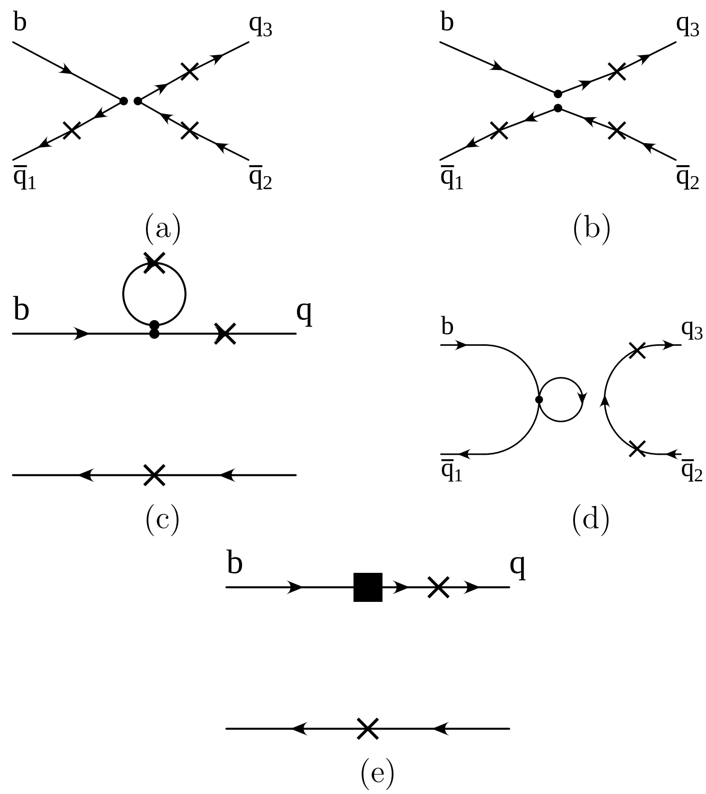

FIG. 1. Quark diagrams contributing to $\bar{B} \to V\gamma$ decays. The cross marks the attachment of the photon line. a) weak annihilation amplitude A; b) W-exchange amplitude E; c) penguin amplitudes $P^{(1)}_{q'}$ (the photon is attached to the $\bar{q} = \bar{d}, \bar{s}$ quark or the quark $q' = u, c$ running in the loop) and $P^{(2)}_{q'}$ with the photon attached to the spectator quark; d) annihilation penguin amplitudes $PA_{q'}$; e) amplitudes with one insertion of the gluonic penguin $G^{(1)}$ (photon attaching to the $\bar{q}$ line) and $G^{(2)}$ (photon attaches to the spectator quark). The box denotes one insertion of the gluonic penguin operator $\mathcal{Q}_8$.